ELECTRICITY AND APPLIED MECHANICS*

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- Elements of physics
- General management
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- Industrial accountancy
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- Economics
- SI metric system of measurement
- Marketing
- Mathematics
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ELECTRICITY

I. SIMPLE CIRCUITS

A. Practical Electrical Units.

When we talk of an electrical system, we use the terms ampere, volt, and ohm.

The **ampere** (A) is a unit of current or rate of flow of electricity.

The **volt** (V) is a unit of potential difference, or electromotive force (emf).

The **ohm** (Ω) is a measure of the resistance.

These three units can be related by Ohm's law which states that:

\[ E = IR \]

where \( E \) is the electromotive force (emf) in volt

\( I \) the current in ampere

\( R \) the resistance in ohm

Another electrical unit, the **coulomb** (C), is related to the ampere by the relationship:

1 ampere = 1 coulomb per second

Thus, we can say that the ampere is a rate of flow of electricity, while the coulomb represents an amount of electricity.

**Example 1.** How much current flows through a wire if 50,000 C pass a certain point every hour?

Current = coulomb/second = C/sec = 50,000 C/3600 sec = 13.9 A

**Example 2.** What voltage will cause 2 A to flow through a resistance of 55 ohms?

According to Ohm's law \( E = IR \)

so \( E = 2\text{A} \times 55\Omega = 110 \text{V} \)
B. Resistivity

The resistance of a conductor depends not only upon its length but also upon its cross-sectional area.

The longer the conductor, the greater the resistance; the thicker the conductor, the less the resistance.

The resistance also depends upon the type of material being used. That is, some materials are better conductors than others, all other conditions being the same.

We can relate resistance to these factors by the formula:

\[ R = \frac{\rho \ell}{A} \]

where \( R \) is the resistance of the conductor in ohm
\( \rho \) (rho) is the specific resistance of the substance expressed in ohm per meter
\( \ell \) is the length in meter
\( A \) is the cross-sectional area in square meter.

**Example 3.** What is the resistance of 200 m of copper wire whose cross-sectional area is \( 5.0 \times 10^{-4} \text{ m}^2 \)?
\( (\rho = 1.7 \times 10^{-8} \text{ ohm meter}) \)

\[ R = \frac{\rho \ell}{A} = \frac{(1.7 \times 10^{-8} \text{ ohm-m}) (200 \text{ m})}{5.0 \times 10^{-6} \text{ m}^2} = 0.68 \text{ ohm} \]

**Example 4.** A tungsten filament \( 1.0 \text{ mm}^2 \) in cross-sectional area is 0.40 m long.

If its resistance is 0.022 ohm, what is its resistivity?

First, we change the cross-sectional area to square meter

\[ 1.0 \text{ mm}^2 = 1.0 \times 10^{-6} \text{ m}^2 \]

Then, since \( R = \frac{\rho \ell}{A} \quad \rho = RA \)

\[ \rho = \frac{(0.022 \text{ ohm}) (1.0 \times 10^{-6} \text{ m}^2)}{0.40 \text{ m}} = 5.5 \times 10^{-8} \text{ ohm-m} \]

C. Resistance and Temperature

The resistance of a conductor depends not only upon its length and cross-sectional area but also upon its temperature.

The resistance of most metals increases with increase in temperature.
This change is similar to that change in the length of a substance as it is heated, or similar to the change in the volume of a gas as it is heated.

The resistance at any temperature can be calculated by the formula

$$ R = R_0 (1 + \alpha t) $$

where $R$ is the resistance of the conductor at temperature $t$,

$R_0$ is the resistance of that conductor at $0^\circ C$,

$\alpha$ is the temperature coefficient of resistance of the substance.

**Example 5.** The resistance of a copper wire at $0^\circ C$ is 200 ohm. What is its resistance at $60^\circ C$?

$\alpha$ for copper is $4.0 \times 10^{-3} $ $\frac{^\circ C}{^\circ C}$

$$ R = R_0 (1 + \alpha t) = 200 \Omega (1 + 4.0 \times 10^{-3} \times 60^\circ C) = 248 \Omega $$

**Example 6.** The resistance of a wire at $0^\circ C$ is 25 ohm. At $100^\circ C$ it is 36 ohm. What is the temperature coefficient of resistance of the material?

Using the formula

$$ R = R_0 (1 + \alpha t) $$

$$ 36 \Omega = 25 \Omega (1 + \alpha \times 100^\circ C) $$

$$ 36 \Omega = 25 \Omega + 25 \Omega \times \alpha \times 100^\circ C $$

$$ 2500 \frac{\Omega}{^\circ C} = 36 \Omega - 25 \Omega = 11 \Omega $$

$$ \frac{11 \Omega}{2500 \Omega \ ^\circ C} = 4.4 \times 10^{-3} \frac{\Omega}{^\circ C} $$

**D. Resistances in Series**

When several resistances are connected in series, the total resistance is equal to the sum of the individual resistances. Thus

$$ R_{total} = R_1 + R_2 + R_3 + R_4 $$
The current in a series circuit is the same in all parts. Thus, in the above circuit, the current flowing through $R_1$ is the same as that flowing through $R_2$, $R_3$, $R_4$.

The voltage across all the resistances is equal to the sum of the voltages across the individual resistances.

Thus, voltage across the total resistance equals voltage across $R_1$ + voltage across $R_2$ + voltage across $R_3$ + voltage across $R_4$.

Example 7. What is the total resistance in following circuit

$$R_T = 50 \text{ ohms}$$

$$R_T = 20 \text{ ohm} + 10 \text{ ohm} + 8 \text{ ohm} + 7 \text{ ohm} + 5 \text{ ohm} = 50$$

Example 8. If the emf applied to the above circuit is 100 volt, what is the current?

We know $E = IR$ and $I = \frac{E}{R}$

$$I = \frac{100 \text{ V}}{50} = 2.0 \text{ A}$$

Note: This current flows through all parts of the series circuit.

Example 9. What is the voltage across each resistance in the above circuit?

The voltage across each resistance depends upon the size of that resistance and also upon the amount of current flowing through it.

Since voltage $E = IR$ we can calculate the voltage (voltage drop) across each resistor.

For 20 ohm resistor, $V = IR = 2.0 \text{ A} \times 20 \text{ ohm} = 40 \text{ V}$

For 10 ohm resistor, $V = IR = 2.0 \text{ A} \times 10 \text{ ohm} = 20 \text{ V}$

For 8 ohm resistor, $V = IR = 2.0 \text{ A} \times 8 \text{ ohm} = 16 \text{ V}$

For 7 ohm resistor, $V = IR = 2.0 \text{ A} \times 7 \text{ ohm} = 14 \text{ V}$

For 5 ohm resistor, $V = IR = 2.0 \text{ A} \times 5 \text{ ohm} = 10 \text{ V}$

We see that the total voltage across the circuit is equal to the sum of the individual voltages across each resistance.
E. Resistances in Parallel.

When resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances,

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \text{ etc.} \]

In a parallel set of resistors, the same emf is applied to each resistor.

In a parallel set of resistors, the sum of the currents in each branch is equal to the main current.

**Example 10.** Three resistances of 20, 10 and 4 ohm are parallel. What is the resistance of this combination?

\[ \frac{1}{R} = \frac{1}{20} + \frac{1}{10} + \frac{1}{4} = \frac{1}{20} + \frac{2}{20} + \frac{5}{20} = \frac{8}{20} \]

\[ R = \frac{20}{8} = 2.5 \text{ ohm} \]

**Example 11.** If the emf across the resistors in example 10 is 10 volt. What is the total current flowing? How much current flows through each resistor?

Since the emf is 10 V and the total resistance is 2.5 ohm, the total current

\[ I = \frac{E}{R} = \frac{10 \text{ V}}{2.5} = 4.0 \text{ A} \]

For the 20 ohm resistor, \( I = \frac{10 \text{ V}}{20} = 0.5 \text{ A} \)

10 ohm \( I = \frac{10 \text{ V}}{10} = 1.0 \text{ A} \)

4 ohm \( I = \frac{10 \text{ V}}{4} = 2.5 \text{ A} \)

Total current 4.0 A

We see that the sum of the currents in each branch is equal to the current flowing through that combination.

**Example 12.** What resistance must be placed in parallel with a 10 ohm resistor to produce a combined resistance of 8 ohm?

Using the formula: \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \)

\[ \frac{1}{R} = \frac{1}{8} + \frac{1}{10} = \frac{5}{40} + \frac{4}{40} = \frac{9}{40} \]

\[ R = \frac{40}{9} \]

R2 = 40 ohm
Example 13. Given the following circuit.

Find:  
- the total resistance
- the total current
- the voltage drop across each resistor
- the current through each branch

\[ R_T = \frac{1}{\frac{1}{20} + \frac{1}{10} + \frac{1}{10}} = \frac{1}{\frac{1}{20} + \frac{2}{20} + \frac{2}{20}} = \frac{5}{20} = \frac{1}{4} \]

\[ R_T = 4 \, \text{ohm} \]

Next, we consider the parallel combination as equivalent to a 4 ohm resistor in the same circuit.

Then, since all the resistors are in series, the total resistance is:

\[ 3 \, \text{ohm} + 5 \, \text{ohm} + 4 \, \text{ohm} = 12 \, \text{ohm} \]

The total current is

\[ I = \frac{E}{R} = \frac{60 \, \text{V}}{12 \, \text{ohm}} = 5.0 \, \text{A} \]

The voltage drop across the 3 ohm resistor = \( IR = 5A \times 3 = 15 \, \text{V} \)

5 ohm = \( 5A \times 5 = 25 \, \text{V} \)

the combination = \( 5A \times 4 = 20 \, \text{V} \)

To find the current flowing through each resistor, the total current (5A) flows through all parts of a series circuit, so that this must be the current flowing through the 3 and 5 ohm resistors, and this must also be the total current flowing through the combination.

For each resistor in the combination \( I = \frac{E}{R} \)

so in the 20 ohm resistor \( I = \frac{20 \, \text{V}}{20 \, \text{ohm}} = 1.0 \, \text{A} \)

\( 20 \, \text{V} = IR \) drop across the combination)
in each of the 10 ohm resistors \( I = \frac{20 \text{ V}}{10 \Omega} = 2.0 \text{ A} \)

Thus the combination circuit becomes

\[ I = 1.0 \text{ A} + 2.0 \text{ A} + 2.0 \text{ A} = 5 \text{ A} \]

**Example 14.** A battery having an emf of 24 volt and an internal resistance of 1 ohm is connected to five 35 ohm resistors in parallel.

Find:
- the total resistance
- the total current
- the current through each resistor
- the terminal voltage of the battery while it is delivering current.

\[ \begin{array}{c|c|c|c|c|c|c} \text{24 Volts} & \text{10 ohms} & \text{35 ohms} & \text{35 ohms} & \text{35 ohms} & \text{35 ohms} & \text{35 ohms} \\ \end{array} \]

a) resistance of the parallel combination,

\[ \frac{1}{R} = \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} + \frac{1}{35} = \frac{5}{35} = \frac{1}{7} \]

\[ R_{\text{paral.comb.}} = 7 \text{ ohm} \]

The total resistance equals the resistance of the parallel combination plus that of the battery

\[ R_{T} = 7 \text{ ohm} + 1 \text{ ohm} = 8 \text{ ohm} \]

b) The total current \( \frac{E}{R} = \frac{24 \text{ V}}{8 \Omega} = 3.0 \text{ A} \)

c) The current through each resistor.

Since all five resistances are the same, \( \frac{1}{8} \) of the total current will flow through each,

or, \( \frac{1}{5} \times 3.0 \text{ A} = 0.6 \text{ A} \) through each resistor.

d) The terminal voltage of the battery equals the emf minus the IR drop due to the internal resistance.

The IR drop in the battery = 3 A x 1Ω = 3 V so that the terminal voltage is 24 V - 3 V = 21 V

Thus the actual IR drop across the parallel resistors is 21 V

Using this figure to find the current through each of the branches, we have

\[ I = \frac{E}{R_{\text{paral.comb.}}} = \frac{21 \text{ V}}{35} = 0.6 \text{ A} \]
II. ELECTRICAL POWER

As in mechanics, power is the rate of doing work.

In an electrical circuit, a certain amount of power is required to drive a motor or to warm a heating unit.

The most common unit of electrical power is the watt \((W)\), which is equivalent to \(\text{1 joule/second}\).

Also, \(\text{1 watt}\) is the power developed when a current of \(\text{1 ampere}\) flows through a resistor with a potential difference of \(\text{1 volt}\) across it.

We can summarize the units as follows:

\[
\begin{align*}
\text{watt} &= \text{volt} \times \text{ampere} = \text{1 VA} \\
\text{1 watt} &= \text{energy/time} = \text{1 joule/sec} = \text{1Nm/sec} \\
\text{1000 watt} &= \text{1 kilowatt} \\
\text{146 watt} &= \text{1 HP (horsepower)}
\end{align*}
\]

Since \(\text{1 ampere} = \text{1 coulomb/sec} \ (\text{1 A} = \text{1 C/sec})\) and since \(\text{watt} = \text{joule/sec} \ (\text{W} = \text{J/sec})\), we can show that:

\[
\begin{align*}
\text{watt} &= \text{volt} \times \text{ampere} = \text{volt} \times \text{coulomb/sec} \\
\text{watt} &= \text{joules/sec} \quad \text{so} \\
\text{joule/sec} &= \text{volts} \times \text{coulomb/sec} \quad \text{and} \\
\text{joule} \ \text{J} &= \text{volt} \times \text{coulomb} = \text{VC}
\end{align*}
\]

III. HEAT ENERGY

When a current passes through a resistor, we can calculate the energy used by means of the above formula:

\[
\text{watt} = \text{volt} \times \text{ampere} = \text{VA} \quad (E \times I)
\]

However, from Ohm's law, we know that:

\[
E = IR
\]

so that, \(\text{watt} = E \times I = IR \times I = I^2R\)

which will tell us the amount of energy consumed by an electrical device.

Next, since \(\text{watt} = \text{joule/sec} \quad \text{joule} = \text{W/sec} = \text{I}^2\text{Rt}\) (where \(t\) is the time in seconds.

We can convert this amount of energy from joule to kilocalorie by using the following conversion:

\[
1 \text{ kcalorie} = 4.186 \times 10^3 \text{ joule}
\]
Example n°1: An electric toaster draws 8.00 ampere from a 115 volt line. How much power does it consume in watt and in kcal/sec?

\[ \text{watt } W = \text{volt } V \times \text{ampere } A = 115 \text{ V} \times 8 \text{ A} = 920 \text{ W} \]

To change watt to kcal/sec, we know that

\[ W = \text{joules/sec} \text{ and also that} \]

\[ 1 \text{ kcal} = 4.186 \times 10^3 \text{ J} \text{ so} \]

\[ 920 \text{ W} = 920 \frac{\text{J}}{\text{sec}} = \frac{920 \text{ J/sec}}{4.186 \times 10^3 \text{ J/kcal}} = 0.220 \text{ kcal/sec} \]

Example n°2: A motor draws 100 A from a 220 V line.

If efficiency is 91%, how much horsepower does it produce?

The power drawn by the motor is \(100 \text{ A} \times 220 \text{ V} = 22,000 \text{ W}\)

If 91% of this energy is available, then the motor produces:

\[ 0.91 \times 22,000 \text{ W} = 20,000 \text{ W} = \frac{20,000 \text{ W}}{746 \text{ W/HP}} = 26.7 \text{ HP} \]

Example n°3: How much power is used by a 5 ohm resistor if it draws 6 A?

Since power = \(I^2R = (6 \text{ A})^2 \times 5 \text{ ohms} = 180 \text{ W}\)

Example n°4: A heating element has a potential difference of 100 V across it and produces 0.500 kcal/sec.

What is its resistance?

First to change kcal/sec to W

\[ 0.500 \frac{\text{kcal}}{\text{sec}} = 0.500 \frac{\text{kcal}}{\text{sec}} \times 4.186 \times 10^3 \frac{\text{J}}{\text{kcal}} = 2050 \frac{\text{J}}{\text{sec}} = 2050 \text{ W} \]

Next, \( I = \frac{W}{V} = \frac{2050 \text{ W}}{100 \text{ V}} = 20.9 \text{ A} \)

Power = \(I^2R\) so \(2050 \text{ W} = (20.9 \text{ A})^2 \times R\)

\[ R = \frac{2050 \text{ W}}{(20.9 \text{ A})^2} = 4.78 \text{ ohm} \]

However, we could have arrived at the same answer by another method without finding the value of the current.

Since \(I = \frac{E}{R}\)

\[ \text{Power} = I^2R = \left(\frac{E}{R}\right)^2 \times R = \frac{E^2}{R} \]

\[ R = \frac{E^2}{\text{power}} \]

\[ R = \frac{(100 \text{ V})^2}{2050 \text{ W}} = 4.78 \text{ ohm} \]
Example no. 5: A 110 V 15 A motor produces 1600 W. What is its efficiency?

How many kcal will be lost if the motor operates 8 hours?

The power input of the motor = 110 V x 15 A = 1650 W

Efficiency = \( \frac{\text{output}}{\text{input}} = \frac{1600 \text{ W}}{1650 \text{ W}} \times 100 = 97\% \)

Power lost as heat = 1650 W - 1600 W = 50 W = 50 J/sec

\( \frac{50 \text{ J/sec}}{4.186 \times 10^3 \text{ J/kcal}} = 1.19 \times 10^{-2} \text{ kcal/sec} \)

\( 1.19 \times 10^{-2} \text{ kcal/sec} \times 8 \text{ hrs} \times 3600 \text{ sec/hr} = 343 \text{ kcal} \)

Example no. 6: A 3 ohm and a 6 ohm resistor are to be connected to a 6 V battery. What is the wattage across each resistor when they are connected in series and then in parallel?

First, in series, the total \( R = 3 \text{ ohm} + 6 \text{ ohm} = 9 \Omega \)

\( I = \frac{E}{R} = \frac{6 \text{ V}}{9} = \frac{2}{3} \text{ amperes} \)

Power produced across 3 ohm resistor = \( I^2R = \left(\frac{2}{3}A\right)^2 \times 3\Omega = 1.33 \text{ W} \)

Power produced across 6 ohm resistor = \( I^2R = \left(\frac{2}{3}A\right)^2 \times 6\Omega = 2.67 \text{ W} \)

Total power produced = 4.00 W

Next in parallel, \( \frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \)

\( R = 2 \text{ ohm} \)

\( I = \frac{E}{R} = \frac{6 \text{ V}}{2} = 3 \text{ A} \)

Power produced across 3 ohm resistor = \( I^2R = \left(3A\right)^2 \times 3\Omega = 27 \text{ W} \)

Power produced across 6 ohm resistor = \( I^2R = \left(3A\right)^2 \times 6\Omega = 54 \text{ W} \)

Total power produced = 81 W
IV. Electrostatics

Many substances, after being rubbed, show an attraction for light objects. When a piece of hard rubber is rubbed with wool, it will attract small pieces of paper and other light substances. This piece of rubber will repel another piece which has been similarly rubbed. Next, if we rub a glass rod with silk and then bring it near the rubber rod, we will notice an attraction between the two. This phenomenon indicates that there are two types of electricity, one positive and the other negative. Like charges will repel each other while opposite charges will attract.

A. COULOMB’S LAW

Coulomb proved that the force exerted by one charge on another charge varies directly as the product of the charges and inversely as the square of the distance between them. This can be stated mathematically as:

$$F = \frac{Q_1 Q_2}{kr^2}$$

where $F$ is the force of attraction or repulsion (in nt), $Q_1$ and $Q_2$ are the two charges (in coulombs), $r$ is the distance between them (in m), and $k$ is the dielectric constant which depends on the medium between the charges. For a vacuum (or for air),

$$k = 1.11 \times 10^{-9} \text{ coulomb}^2/\text{nt-m}^2$$

**Example No. 1.** Two charges of +5.0 microcoulombs are .50 m apart in air. What is the force of repulsion between them?

$$F = \frac{Q_1 Q_2}{kr^2} = \frac{(5.0 \times 10^{-9} \text{ coul}) (5.0 \times 10^{-9} \text{ coul})}{(1.11 \times 10^{-9} \text{ coul}^2/\text{nt-m}^2)(.50 \text{ m})^2} = .90 \text{ nt}$$

**Example No. 2.** Two small pith balls are suspended one above the other at a distance of .050 m. The lower one has a charge of +2.0 micromicrocoulombs and a mass of $2.5 \times 10^{-4}$ kg. What must be the charge on the upper ball in order to lift the lower one?

From Coulomb’s law, $Q_i = (Fkr^2)/Q_2$. From Newton’s second law, $F = ma$.

Combining these two formulas:

$$Q_i = \frac{ma r^2}{Q_2} = \frac{(2.5 \times 10^{-4} \text{ kg} \times 9.80 \text{ m/sec}^2)(1.11 \times 10^{-9} \text{ coul}^2/\text{nt-m}^2)(.050 \text{ m})^2}{2.0 \times 10^{-6} \text{ coul}}$$

$$= 340 \text{ microcoulombs}$$

which must be a negative charge in order to attract the other charge which is positive.
Example No. 3. Three $+50$-microcoulomb charges are located on a meter stick at positions $0$, $.20$, and $.50$ m, respectively. What force is exerted by the first two charges on the third? What is the force exerted by the two end charges on the one in the middle?

\[ +50 \mu \text{COUL} \quad +50 \mu \text{COUL} \quad +50 \mu \text{COUL} \]

\[ 0 \quad .20 \text{ M} \quad .50 \text{ M} \]

The charge at the $0$ mark will exert a force of repulsion on the charge at the $.50$ m mark since the charges are alike. This force will be

\[ F = \frac{Q_1Q_2}{kr^2} = \frac{(50 \times 10^{-6} \text{ coul}) (50 \times 10^{-6} \text{ coul})}{(1.11 \times 10^{-10} \text{ coul}^2/\text{nt-m}^2) (.50 \text{ m})^2} = 90 \text{ nt} \]

Likewise, the charge at the $.20$-m mark will exert a force of repulsion on the charge at the $.50$-m mark. This force will be

\[ F = \frac{Q_1Q_2}{kr^2} = \frac{(50 \times 10^{-6} \text{ coul}) (50 \times 10^{-6} \text{ coul})}{(1.11 \times 10^{-10} \text{ coul}^2/\text{nt-m}^2) (.20 \text{ m})^2} = 250 \text{ nt} \]

The total force exerted on the charge at the $.50$-m mark will be $90 \text{ nt} + 250 \text{ nt} = 340 \text{ nt}$ repulsion. We note that we can add the forces directly since they are acting in a straight line rather than at angles with each other.

Next, to calculate the force exerted on the center charge by the charge at the $0$ mark,

\[ F = \frac{Q_1Q_3}{kr^2} = \frac{(50 \times 10^{-6} \text{ coul}) (50 \times 10^{-6} \text{ coul})}{(1.11 \times 10^{-10} \text{ coul}^2/\text{nt-m}^2) (.20 \text{ m})^2} = 563 \text{ nt} \]

This means that the charge at the $0$ mark will exert a force of repulsion of $563 \text{ nt}$ on the charge at the $.20$-m mark and will act toward the right. We have already calculated that the charge on the $.20$-m mark is acted on by a force of $250 \text{ nt}$ by the charge at the $.50$-m mark, so that the charge at the $.50$-m mark will exert a force of repulsion of $250 \text{ nt}$ on the charge at the $.20$-m mark, this force acting toward the left. Thus we have two linear forces acting on the charge at the $.20$-m mark, one with a force of $563 \text{ nt}$ acting toward the right and the other with a force of $250 \text{ nt}$ acting toward the left. The resultant force will be the difference, or a net force of $313 \text{ nt}$ acting toward the right.

Example No. 4. Three charges of $+3.0$, $-3.0$, and $+5.0$ microcoulombs, respectively, are placed at the vertices of an equilateral triangle $ABC$ having sides $.050$ m long. What is the magnitude and the direction of the resultant force acting on the $+5.0$-microcoulomb charge?

The force of repulsion between the $+3.0$-microcoulomb charge and the $-5.0$-microcoulomb charge can be calculated from

\[ F = \frac{Q_1Q_2}{kr^2} = \frac{(3.0 \times 10^{-6} \text{ coul}) (5.0 \times 10^{-6} \text{ coul})}{(1.11 \times 10^{-10} \text{ coul}^2/\text{nt-m}^2) (.050 \text{ m})^2} = 54 \text{ nt} \]

The force of attraction between the $-3.0$-microcoulomb charge and the $+5.0$ microcoulomb charge can also be calculated to be $54 \text{ nt}$ (since the charges are the same except for sign, and the distance is the same).

The $-3.0$-microcoulomb charge at $A$ will exert a force of $54 \text{ nt}$ on the charge at $C$ along path $CD$.

The $-3.0$-microcoulomb charge at $B$ will exert a force of $54 \text{ nt}$ on the charge at $C$ along path $CE$. 

To find the resultant of these forces, we complete the parallelogram of the forces and have the diagram at the right with the resultant force acting along path CF.

Triangle CEF is an equilateral triangle because it is similar to triangle ABC (given as an equilateral triangle) since the sides of the two triangles are parallel to each other. Thus, since CEF is an equilateral triangle, the resultant force CP is 54 nt in the indicated direction.

**Example No. 5.** Two small objects have charges of \(+4 \times 10^{-4}\) and \(-16 \times 10^{-4}\) coulombs, respectively. What is the force of attraction between them if they are .10 m apart in air? If the objects are touched and then returned to their original position, what will be the force between them?

First, the force of attraction between the two objects is

\[
F = \frac{Q_1Q_2}{k r^2} = \frac{(4 \times 10^{-4}\text{ coul}) (16 \times 10^{-4}\text{ coul})}{(1.11 \times 10^{-19}\text{ coul}^2/\text{nt-m}^2) (.10\text{ m})^2} = 58.2\text{ nt}
\]

Next, if the objects touch, the resultant charge will be \(-12 \times 10^{-4}\) coul \((+4 \times 10^{-4}\text{ coul}) + (-16 \times 10^{-4}\text{ coul})\) with a net charge of \(-6 \times 10^{-4}\) coul on each. Then, the force between them (repulsion since both charges are negative) is

\[
F = \frac{Q_1Q_2}{k r^2} = \frac{(6 \times 10^{-4}\text{ coul}) (6 \times 10^{-4}\text{ coul})}{(1.11 \times 10^{-19}\text{ coul}^2/\text{nt-m}^2) (.10\text{ m})^2} = 32.7\text{ nt}
\]

**B. FIELD INTENSITY**

The space around a charged body is called the field of that body. The field intensity \((E)\) is a vector quantity and can be defined as the force acting on a unit charge at a distance \(r\) from a charge \(Q\) in a medium of dielectric constant \(k\), or

\[
E = \frac{Q}{kr^2}
\]

If \(Q\) is in coulombs and \(r\) in meters, \(E\) will be in newtons per coulomb. By comparing this formula with Coulomb’s law, we see that

\[
E = F/Q \quad \text{since} \quad F = \frac{Q_1Q_2}{kr^2} \quad \text{and} \quad \frac{F}{Q} = \frac{Q_1Q_2}{kr^2} = \frac{Q}{kr^2}
\]

We can rearrange this formula to show that \(F = EQ\), or the force acting on a charge \(Q\) at a point where the field intensity is \(E\) is equal to \(QE\).

**Example No. 6.** What is the field intensity midway between a charge of \(+20\) microcoulombs and a charge of \(-5.0\) microcoulombs separated by a distance of .50 m?

\[+50\mu\text{ coul} \quad \underline{ \quad .25\text{ m} \quad .25\text{ m} \quad -5.0\mu\text{ coul} }\]

If we select a point halfway between the two charges and assume a charge of \(+1\) microcoulomb to be located there, it will be repelled by the \(+20\)-microcoulomb charge and attracted by the \(-5.0\)-microcoulomb charge.

The force of repulsion \[\frac{Q_1Q_2}{kr^2} = \frac{(20 \times 10^{-4}\text{ coul}) (1 \times 10^{-4}\text{ coul})}{(1.11 \times 10^{-19}\text{ coul}^2/\text{nt-m}^2) (.25\text{ m})^2} = 2.91\text{ nt to the right}\]

The force of attraction \[\frac{Q_1Q_2}{kr^2} = \frac{(2.0 \times 10^{-4}\text{ coul}) (1 \times 10^{-4}\text{ coul})}{(1.11 \times 10^{-19}\text{ coul}^2/\text{nt-m}^2) (.25\text{ m})^2} = .73\text{ nt to the right}\]

so that the resultant force is 2.91 nt + .73 nt, or 3.64 nt to the right. The field intensity is \(F/Q\) and since we selected a 1-microcoulomb charge at the midpoint,

\[
\text{The field intensity } E = \frac{3.64\text{ nt}}{1 \times 10^{-4}\text{ coul}} = 3.64 \times 10^4\text{ nt/coul}
\]
EXAMPLE No. 7. What would be the force on a charge of +10 microcoulombs placed halfway between the charges in example No. 6?

Since \( F = EQ \), \( F = 5.6 \times 10^4 \text{ nt/coul} \times 10 \times 10^{-4} \text{ coul} = 36.4 \text{ nt} \)

EXAMPLE No. 8. A force of 15 nt is required to hold a charge of 100 microcoulombs at a certain point. What is the field strength at that point?

Since \( E = F/Q \),

\[
E = \frac{15 \text{ nt}}{100 \text{ microcoulombs}} = \frac{15 \text{ nt}}{100 \times 10^{-4} \text{ coul}} = 15 \times 10^4 \text{ nt/coul}
\]

C. POTENTIAL DIFFERENCE

The potential at any point, due to a charge, is the work done in bringing a unit positive charge from an infinite distance up to that point against the forces of the field. The potential at a point is given by the formula

\[
V = \frac{Q}{kr}
\]

where \( V \) is in volts when \( Q \) is in coulombs, \( k \) is in \( \text{coul}^2/\text{nt-m}^2 \), and \( r \) is in meters

\[
V = \frac{Q}{kr} = \frac{\text{coul}}{\text{coul}^2/\text{nt-m}^2 \times \text{m}} = \frac{\text{nt-m}}{\text{coul}} = \frac{\text{joules}}{\text{coul}} = \text{volts}
\]

The potential difference between two points is defined as the work required to transfer positive electricity from one point to another. The work done in transferring a charge from one point to another point having a potential difference of \( V \) is given by the formula

\[
W = QV
\]

where \( W \) is in joules when \( Q \) is in coulombs and \( V \) in volts.

Note: Both potential and work are scalar quantities and, as such, depend only upon distances and not upon direction from charges.

EXAMPLE No. 9. What is the potential .50 m from a charge of 200 microcoulombs?

Since \( V = Q/kr \)

\[
V = \frac{200 \times 10^{-4} \text{ coul}}{(1.1 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (0.50 \text{ m})} = 7.2 \times 10^4 \text{ nt-m/coul} = 7.2 \times 10^4 \text{ volts}
\]

EXAMPLE No. 10. What is the potential difference midway between two charges 2.0 m apart if the charges are: (a) +1500 and +1500 micromicrocoulombs; (b) +1500 and -1500 micromicrocoulombs; (c) +1500 and -1000 micromicrocoulombs.

(a) Since both charges are the same, the potential will be twice that due to one charge.

Potential due to +1500 micromicrocoulomb charge = \( Q/kr \), so

\[
V = \frac{1500 \times 10^{-12} \text{ coul}}{(1.1 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (1.0 \text{ m})} = 13.6 \text{ nt-m/coul} = 13.6 \text{ volts}
\]

The potential difference due to the other +1500-micromicrocoulomb charge is the same, so that the total potential at the mid-point is

\[ 13.6 \text{ volts} + 13.6 \text{ volts} = 27.2 \text{ volts} \]
(b) The potentials due to the 1500-micromicrocoulomb charges are the same as above except that here one is positive and the other negative, so that the potential at the mid-point is

\[ 13.6 \text{ volts} - 13.6 \text{ volts} = 0 \text{ volt} \]

(c) Here again the potential due to the +1500-micromicrocoulomb charge is 13.6 volts. The potential due to the -1000-micromicrocoulomb charge is

\[ V = \frac{-1000 \times 10^{-12} \text{ coul}}{(1.1 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (1.0 \text{ m})} = -9.09 \text{ volts} \]

so that the potential at the mid-point = 13.6 volts − 9.09 volts = 4.5 volts.

**Example No. 11.** What is the potential at point \( X \) which is equidistant from two charges of +4.0 and +5.0 microcoulombs and .30 m from the mid-point of the line joining them if the charges are .80 m apart?

Using the Pythagorean theorem,

\[ (XA)^2 = (XC)^2 + (AC)^2 = (.30 \text{ m})^2 + (.40 \text{ m})^2 = .25 \text{ m}^2 \]

\[ XA = .50 \text{ m} \]

Then, the total potential at \( X \) is the sum of the potentials due to the separate charges.

Potential due to +4.0 microcoulomb charge \( Q/kr = \frac{4.0 \times 10^{-4} \text{ coul}}{(1.11 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (.50 \text{ m})} = 7.2 \times 10^4 \text{ nt-m/coul} = 7.2 \times 10^4 \text{ volts} \]

Potential due to +5.0-microcoulomb charge \( Q/kr = \frac{5.0 \times 10^{-4} \text{ coul}}{(1.11 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (.50 \text{ m})} = 9.0 \times 10^4 \text{ nt-m/coul} = 9.0 \times 10^4 \text{ volts} \]

Total Potential = 16.2 \times 10^4 \text{ volts} = 1.62 \times 10^5 \text{ volts}  

**Example No. 12.** A dry cell has a potential difference of 1.5 volts between its terminals. How much work is required to transfer 10.0 coul from one terminal to the other?

Again, \( W = QV = 10.0 \text{ coul} \times 1.5 \text{ volts} \), and since volts = joules/coul,

\[ W = 10.0 \text{ coul} \times 1.5 \text{ joules/coul} = 15 \text{ joules} \]

**Example No. 13.** How much work is required to carry a charge of +500 microcoulombs from a point 20 m from a charge of +1000 microcoulombs to a point 2.0 m from it?

Since work = charge \( \times \) potential difference, we must find the potential difference between the two points.

At a point 20 m from the +1000 \( \mu \)coul charge, \( V = \frac{Q/kr = +1000 \times 10^{-12} \text{ coul}}{(1.1 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (20 \text{ m})} = .45 \text{ nt/coul} = .45 \text{ volt} \)

At a point 2.0 m from the +1000-\( \mu \)coul charge, \( V = \frac{Q/kr = +1000 \times 10^{-12} \text{ coul}}{(1.1 \times 10^{-19} \text{ coul}^2/\text{nt-m}^2) (2.0 \text{ m})} = 4.5 \text{ nt/coul} = 4.5 \text{ volts} \)

Therefore, the potential difference between these two points is

\[ 4.5 \text{ volts} - .45 \text{ volt} = 4.05 \text{ volts} \]

Then, work \( = QV = +500 \mu \text{coul} \times 4.05 \text{ volts} = 500 \times 10^{-12} \text{ coul} \times 4.05 \text{ joules/coul} = 2.0 \times 10^{-4} \text{ joule} \)
V. Meters

1. The Galvanometer

The most commonly used galvanometer consists of a coil pivoted between the poles of a magnet.

When a current flows through the coil, it is deflected and moves a pointer connected to it.

The pointer moves across a scale, and the amount of deflection on the scale is proportional to the amount of current flowing through the coil.

The motion of the coil is opposed by a spring which returns the coil and the pointer to the original position when the current is shut off.

2. The Voltmeter

A voltmeter is simply a galvanometer with a resistance connected in series with its coil.

The deflection is proportional to the applied voltage and inversely proportional to the resistance of the coil and the additional resistor.

By selecting the proper size external resistor, we can make the pointer indicate the applied voltage. In general, a voltmeter has a very high external resistance connected to it.

A voltmeter is always connected in parallel with the resistance across which the voltage is to be measured.
3. **The Ammeter**

An ammeter is simply a galvanometer with a by-pass resistor connected in parallel with the coil.

As with a voltmeter, the resistance of the by-pass may be selected so that the reading on the scale is equal to the total current in the circuit.

In general the by-pass resistor, or the shunt, has a very low resistance, so that the amount of current flowing through the coil is very small compared to that through the shunt itself. (In the case of the voltmeter, the high resistance connected in series also serves to limit the current flowing through the coil).

An ammeter is always connected in series with the current to be measured.

**Example No. 1:** A galvanometer coil has a resistance of 30.0 ohms. A current of 0.050 amp. gives a full scale deflection. What external resistance must be connected in series with the coil so that its range will be 100 volts?

Since the voltage applied will be 100 volts (or less), and since the maximum current is 0.050 amp., we can find the total resistance by applying Ohm's law. So,

\[ R = \frac{V}{I} = \frac{100 \text{ volts}}{0.050 \text{ amp.}} = 2000 \text{ ohms} \]

However, this is the total resistance to be connected to the 100 volt source. The coil itself has a resistance of 30 ohms, so that the additional resistance must be 2000 - 30 or 1970 ohm.

**Example No. 2** A galvanometer coil has a resistance of 25 ohm. A current of 100A causes a full-scale deflection.

What size shunt is required to convert this galvanometer to an ammeter which will indicate full-scale deflection for a current of 5.00A?

Since the current through the coil is 100A, the rest of the current must flow through the shunt.

Thus the current through the is 5.00A - 100A, or 4.90A.

The shunt and the coil are connected in parallel, so that the voltage drop across each is the same. Thus,
\[ E_{\text{across coil}} = IR = 0.1 \text{ A} \times 25 \text{ ohm} \]

\[ E_{\text{across shunt}} = IR = 4.9 \text{ A} \times R \]

Equating these two, we have:

\[ 0.1 \text{ A} \times 25 \text{ ohm} = 4.9 \text{ A} \times R \]

\[ R = \frac{0.1 \text{ A} \times 25 \text{ ohm}}{4.9 \text{ A}} = 0.51 \text{ ohm} \]

Example No. 3: An ammeter has a resistance of .0020 ohm and reads 1 milliampere (mA) per scale division.

What size resistance must be connected as an additional shunt so that the scale will read 20 mA per division?

Since there will be 20 mA through the circuit and only 1 mA will go through the ammeter, there must be 19 mA flowing through the additional shunt.

Then as above, the voltage across the ammeter and the shunt are the same since they are in parallel, and thus their IR drops are the same. So.

Potential drop across ammeter = potential drop across shunt

\[ 1 \text{ mA} \times .0020 \text{ ohm} = 19 \text{ mA} \times R \]

\[ R = \frac{1 \text{ mA} \times .0020 \text{ ohm}}{19 \text{ mA}} = .000105 \text{ ohm} \]

Note: It was not necessary to convert milliamperes to amperes since the units cancelled.

Example No. 4: A voltmeter has a resistance of 5000 ohm and registers 10 volt per scale division.

What size resistor must be connected to this meter to make it read 100 volt per division?

To increase the range of the meter we must connect a resistor in series with it.

When they are connected as shown in the diagram, there will be a potential drop of 10 volt across the voltmeter and a potential drop of 90 volt across the other resistor to give a total drop of 100 volt across the circuit.

Next, since both the meter and the resistor are connected in series, the same current flows through each.
Thus we can say that:

\[ E_{\text{across meter}} = IR = I \times 5000 \text{ ohm} = 10V \]

\[ E_{\text{across resistor}} = IR = 90 \text{ V} \]

Solving for the currents in each case,

\[ I = \frac{10}{5000} \quad \text{and} \quad I = \frac{90}{R}. \]

Equating these, since the current is the same,

\[ \frac{10}{5000} = \frac{90}{R} \]

\[ R = \frac{90 \times 5000}{10} = 45000 \text{ ohm} \]

Thus, a 45000 ohm resistor should be connected in series with the voltmeter to increase the range from 10 V division to 100 V division.

**Example n° 5.** An ammeter has a resistance of 0.1 ohm and its shunt has a resistance of 0.00005 ohm.

What current will flow through the meter when the total current is 2.00 A?

If we call the current through the meter \( I \), then the current through the shunt will be \( 2.00 - I \).

Next, since they are connected in parallel, the potential drop across each is the same, so that the IR are equal.

Thus,

\[ IR \text{ (meter)} = IR \text{ (shunt)} \]

\[ I \times 0.100 = (2.00 - I) \times 0.00005 \]

\[ 0.100 - I = 0.00005 - 0.00005 I \]

\[ 0.100005 I = 0.00001 \]

\[ I = \frac{0.00001}{0.100005} = 0.00009995 \]

\[ I = 0.001 \text{ A} \]

**Example n° 6.** An ammeter of negligible resistance is connected as shown in the diagram.

What current will it indicate when a 1000 ohm voltmeter is connected across the 500 ohm resistor?

What will be the current if a 20,000 ohm voltmeter is used?

When the 1000 ohm voltmeter is used, there are two resistors in parallel: a 500 ohm and a 1000 ohm one.

Their combined resistance in parallel can be found by

\[ \frac{1}{R} = \frac{1}{500} + \frac{1}{1000} = \frac{2}{1000} + \frac{1}{1000} = \frac{3}{1000} \]

\[ R = \frac{1000}{3} = 333 \text{ ohm} \]
Then, \( I = \frac{E}{R} = \frac{10 \ V}{3332 \ \Omega} = 0.3 \ A \)

When the 20,000 ohm meter is used,
\[
\frac{1}{R} = \frac{1}{500} + \frac{1}{300} = \frac{40}{20,000} + \frac{1}{20,000} = \frac{41}{20,000}
\]

\[ R = \frac{20,000}{41} = 488 \ \text{ohm} \]

\[ I = \frac{E}{R} = \frac{10 \ V}{488 \ \text{ohm}} = 0.0205 \ A \]

**Note:** The current should have been \( \frac{E}{R} \) or \( \frac{10 \ V}{500 \ \text{ohm}} = 0.2 \ A \).

We could have obtained a more correct reading by using a higher resistance voltmeter.

![Diagram](image)
APPLIED MECHANICS

HYDRAULIC SWING ARM DIE CUTTING MACHINE

1. The Machine

This machine is a hydraulically operated-electrically actuated press, designed to cut leather between the stationary anvil and the movable press.

1. Main Switch
2. Elec. Control box
3. Motor and Hydraulic Equipment
4. Dial Disk
5. Push Button Handle
6. Double acting Cylinder
2. **Operation**

   The operation of this machine is very simple, because only one push-button must be pressed for stroke lengths up to 12 mm. Two push-buttons are required for stroke lengths between 15 to 60 mm.

3. **Hydraulic stroke length and daylight Adjustment**

   ![Diagram of dial and indicator line]

   The dial disk is for the stroke length and a knob for setting the lowest point of stroke.

   **Note:** Both controls should be operated only when the motor is running.

   The length of stroke is set by pulling out and rotating the stroke dial disk to match the stroke length markings on the disk with the indicator line on the disk housing. Then release the disk to engage the detent mechanism. The swing arm can be moved up or down accordingly.

   The daylight and the lowest point of stroke is set on the right hand dial knob. Counter-clockwise rotation will lower, and clockwise rotation will raise the swing arm. With this adjustment, the lowest point of stroke can be set within 1/10 mm. The swing arm will move up or down as the knob is rotated.
Precautions: The daylight at the lowest point of stroke must always be adjusted from "up to down" - meaning that for resetting when changing from a shallow to a high die the lowest point of the swing arm must always be higher than the die, to be lowered gradually until the arm makes contact with the dies.

Machine Specifications:

Model : 8L
Cutting force : 18,000 kp
Motor : 2.2 kW

Figure 3: Hydraulic Diagram